Tanta University
Faculty of Engineering
Production Engineering Department

Mid-Term Exam (March, 2005)
Second Year, Heat Transfer
Time allowed 1 hour

Answer the following questions. "Neat and clear answers are appreciated"

Question No. 1:

a) What is meant by thermal conductivity of a material. What is the possible factors which may affect the decision of choice an insulating material for an engineering application.

b) The construction of a furnace wall consists of two layers: 7.5 cm fire brick of thermal conductivity 1.1 Wlm°C and 0.64 cm mild steel plate of thermal conductivity 39 Wlm°C. The temperature of the inside surface of the wall is kept constant at 920 K while the temperature of the surrounding air is 300K. If the heat transfer coefficient from the outside surface of the wall is 68 Wlm^2 °C, determine the heat transfer losses per unit area of the wall and the outside surface temperature.

Question No. 2:

a) Heat is uniformly generated inside a solid circular rod by the rate of q_v Wlm^3. The length of the rod is long enough such that all of the generated heat is considered to diffuse to the outer surface of the rod in the radial direction. Starting from the general equation of heat conduction in cylindrical coordinates:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \Phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_v}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

deduce an expression for the temperature distribution inside the rod. Schematically draw this distribution and show that the maximum temperature inside the rod is expressed as the following:

$$T_{max} = T_w + \frac{q_v R^2}{4k}$$

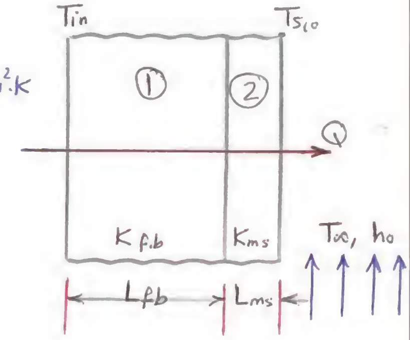
Where, T_w is the outer surface temperature of the rod
R & k are the radius and thermal conductivity of the rod respectively.

b) An electric wire has a diameter of 10 mm, thermal conductivity of 144 Wlm°C and electrical resistance of 0.1 Ω/m. The wire is uniformly covered by an insulating layer of 3 mm thickness and 0.15 W/m°C thermal conductivity. The wire is used in an environment of temperature 30 °C and the heat transfer coefficient from the outside surface of the insulating layer is 86 Wlm^2 °C. If the maximum operating temperature inside the wire should not exceed 150 °C, what is the maximum safe electric current may flow through the wire?.

With best wishes Dr. Kh. khodary

① (1) ⓑ

given :- Composite plane Wall



$T_{in} = 920\,K \quad T_o = 300\,K \quad h_o = 68\ W/m^2.K$

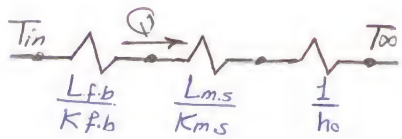
$L_{f.b} = 0.075\ m \quad L_{m.s} = 0.0064\ m$

$K_{f.b} = 1.1\ W/m.K \quad K_{m.s} = 39\ W/m\,K$

Req :- ① $q = \dfrac{Q}{A}$ ② $T_{s,o}$

Soln

$$q = \frac{Q}{A} = \frac{\Delta T}{\sum R_{th}}$$



$$\Delta T = T_{in} - T_\infty = 920 - 300 = 620\ K$$

$$\sum R_{th} = R_{cond①} + R_{cond②} + R_{conv}$$

$$= \frac{L_{f.b}}{K_{f.b}} + \frac{L_{m.s}}{K_{m.s}} + \frac{1}{h_o}$$

$$= \frac{0.075}{1.1} + \frac{0.0064}{39} + \frac{1}{68} = 0.08305\ \frac{K.m^2}{W}$$

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{620}{0.08305} = 7465.2\ W/m^2$$

for steady state heat transfer ⇒ q is the same through all layers

$$q = \frac{T_{in} - T_\infty}{\frac{L_{fb}}{K_{f.b}} + \frac{L_{m.s}}{K_{m.s}} + \frac{1}{h_o}} = \frac{T_{in} - T_{s,o}}{\frac{L_{f.b}}{K_{f.b}} + \frac{L_{ms}}{K_{m.s}}}$$

$$T_{s,o} = T_{in} - q\left(\frac{L_{f.b}}{K_{fb}} + \frac{L_{m.s}}{K_{m.s}}\right)$$

$$= 920 - 7465.2\left(\frac{0.075}{1.1} + \frac{0.0064}{39}\right)$$

$$T_{s,o} = 920 - 510.216 \Rightarrow \boxed{T_{s,o} = 409.784\ K}$$

Exam

Question No. 2:

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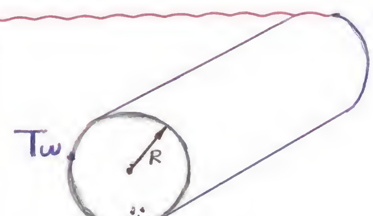
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\underbrace{\frac{\partial^2 T}{\partial \varphi^2}}_{(0)\,①} + \underbrace{\frac{\partial^2 T}{\partial z^2}}_{(0)\,①} + \frac{q_v}{K} = \frac{1}{\alpha}\underbrace{\frac{\partial T}{\partial \tau}}_{(0)\,②}$$

assumption

① one-direction H.T ② steady state H.T

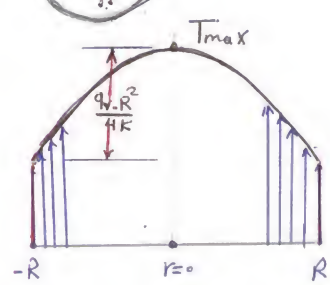


$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{q_v}{K} \xrightarrow{*r} r\cdot\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r} = -\frac{q_v \cdot r}{k}$$

$$\frac{\partial}{\partial r}\left(r\cdot\frac{\partial T}{\partial r}\right) = -\frac{q_v \cdot r}{K} \xrightarrow{\text{بالتكامل}} r\cdot\frac{dT}{dr} = -\frac{q_v \cdot r^2}{2K} + C_1$$

$$\frac{dT}{dr} = -\frac{q_v \cdot r}{2K} + \frac{C_1}{r} \rightarrow \text{I} \xrightarrow{\text{بالتكامل}}$$

$$T = -\frac{q_v \cdot r^2}{4K} + C_1 \ln r + C_2 \rightarrow \text{II}$$



* Temprature distrub- through solid cylinder

B. Conditions

At $r=0 \Rightarrow \left(\frac{dT}{dr}=0\ ;\ T=T_{max}\ ;\ T\text{ is limited}\right) \Rightarrow (C_1=0)$

At $r=R \Rightarrow T=T_w \Rightarrow T_w = -\frac{q_v \cdot R^2}{4K} + C_2 \Rightarrow C_2 = T_w + \frac{q_v \cdot R^2}{4K}$

$$T = -\frac{q_v \cdot r^2}{4K} + T_w + \frac{q_v \cdot R^2}{4K} \Rightarrow \boxed{T = T_w + \frac{q_v \cdot R^2}{4K}\left(1-\left(\frac{r}{R}\right)^2\right)}$$

an expression for Temprature distribution

At $r=0 \Rightarrow T=T_{max} \rightarrow$ sub in eqn III we obtain

$$\boxed{T_{max} = T_w + \frac{q_v \cdot R^2}{4K}}$$